Perceiving Geometric Patterns: From Spirals to Inside–Outside Relations

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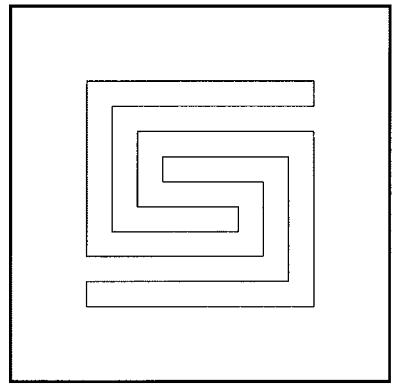
***Abstract*—Since first proposed by Minsky and Papert, the spiral problem is well known in neural networks. It receives much attention as a benchmark for various learning algorithms. Unlike previous work that emphasizes learning, we approach the problem from a different perspective. We point out that the spiral problem is intrinsically connected to the inside–outside problem proposed by Ullman. We propose a solution to both problems based on oscillatory correlation using a time-delay network. Our simulation results are qualitatively consistent with human performance, and we interpret human limitations in terms of synchrony and time delays. As a special case, our network without time delays can always distinguish these figures regardless of shape, position, size, and orientation.**

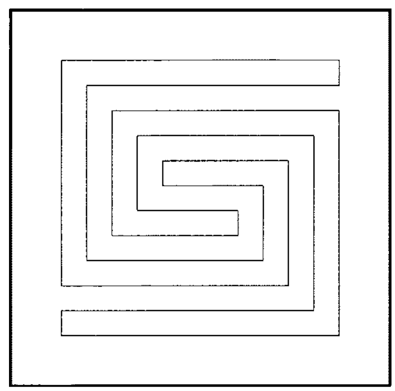
***Index Terms*—Desynchronization, geometric patterns, inside–outside relations, LEGION, oscillatory correlation, spiral problem, synchronization, time delays, visual perception.**

I. INTRODUCTION

THE SPIRAL problem refers to distinguishing between a connected single spiral and disconnected double spirals on a two-dimensional plane, as illustrated in Fig. 1. Minsky and Papert first introduced the problem in their influential 1969 book on perceptrons [29], and the problem belongs to a class of geometric patterns systematically studied in the book. As pointed out by Minsky and Papert [30], humans are unable to solve the problem immediately, but can do it through the use of some serial mental process. Since its introduction, the spiral problem has received much attention from several different fields. In the neural-network community, a variant of the problem has become a benchmark [10] to evaluate the performance of a learning system on both nonlinear separability and generalization, since Lang and Witbrock [25] reported that the problem could not be solved with a standard multilayered perceptron. Many papers have dealt with the problem and many solutions have been attempted using different learning models [2], [5], [7], [11], [13], [25]. However, resulting learning systems are only able to produce decision regions highly constrained by the spirals defined in a training set, thus specific in shape, position, size, orientation, etc. The problem in the geometric form is still open for neural-network learning. Furthermore, no explanation is provided as to why the problem is difficult for human subjects to solve. On the other hand, the problem is considered as an instance of an important stage in visual perception. This is the process whereby a figure, or object, in a scene is separated from other figures and background clutter, so-called *figure–ground separation*, in perceptual psychology. Figure–ground separation is ubiquitous and immediate in general. However, the spiral problem seems to be an exceptional case since it is difficult for humans to give a solution immediately. Therefore, spirals are often viewed as anomalous perceptual stimuli [18]. Grossberg and Wyse [17] proposed a neural-network architecture for figure-ground separation of connected scenic figures based on their earlier



(a)



(b)

Fig. 1. The spiral problem. (a) A connected single spiral. (b) Disconnected double spirals (adapted from [29] and [30]).

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work called the FACADE theory [16]. It was reported that their architecture can distinguish between connected and disconnected spirals. In their paper, however, no demonstration was given to the spiral problem. In the framework of cellular neural networks (CNNs), Chua [8] demonstrated that a connected image component can be distinguished from disconnected ones. Both models, lacking a global control mechanism, encounter difficulty in separating multiple components, and they do not exhibit the limitations that humans do.

A related operation, called curve tracing, has been studied in psychological experiments, whereby subjects detect whether two dots are on the same curve. The main result from a number of studies is that the time to carry out the operation increases monotonically, and roughly linearly beyond a certain length, with the separation of the two dots along the curve [20], [21], [38]. This result suggests that immediate detection is available for short dot separations along a curve but sequential tracing along the curve is employed for long separations. These findings support the notion that humans employ some sequential operation in solving the spiral problem [30]. Interestingly, in the case of apparent sequential tracing, subjects are unaware of the tracing process [38]. Further experiments on curve tracing reveal that the speed of tracing decreases systematically when curve curvature increases, and tracing time is, to a considerable extent, invariant to the absolute size of the pattern [19], [21].

There is a related problem in the study of visual perception, i.e., the perception of spatial relations. For the human visual system, the perception of spatial properties and relations appears to be immediate and effortless, and the response is usually fast and accurate [46]. However, the immediate perception of spatial relations is subject to some limitations. For instance, perceiving the *inside–outside relation* is one such task. Considering the visual input of a single closed curve, the task of perceiving the inside–outside relation is to determine whether a specific pixel lies inside or outside the closed curve. The perception of the inside–outside relation is usually effortless for humans [46]. However, the immediate perception is not available when the bounding contour becomes highly convoluted [43], [44]. In fact, one can make the bounding contour as convoluted as a spiral. Ullman [43] suggested the computation of spatial relations through the use of visual routines. Visual routines result in the conjecture that the inside-outside is inherently sequential. As pointed out recently by Ullman [44], the processes underlying the perception of inside-outside relations are as yet unknown and applying visual routines is simply one alternative.

There are many cortex areas in the mammalian visual system and an everyday object will excite cells in most of these areas. In vision, therefore, binding is viewed as a way of integrating fragmentary neural events at multiple locations in the brain to produce unified perceptual experience and behavior. Binding implies that with activity spread across the cortex areas, visual perception cannot occur until the activity associated with one object can be labeled somehow so as to make those relevant neurons act like a functional unit [35]. Theoretical investigations of brain functions indicate that timing of neuronal activity is a key to the construction of neuronal assemblies [27], [47]. In particular, a binding mechanism that uses neural oscillations to encode temporal correlation has received considerable support biologically. The main discovery of these experiments can be briefly summarized as follows [23], [40]: 1) neural oscillations ranging from 30 to 70 Hz (often referred to as 40 Hz oscillations) are triggered by sensory stimulation; 2) synchronous oscillations occur across an extended brain region if the stimulus constitutes a coherent object; and 3) no phase locking exists across regions stimulated by different stimuli if they are not related to each other.

The discovery of synchronous oscillations in the visual cortex has triggered much interest to develop computational models for oscillatory correlation. Recently, Terman and Wang proposed a *locally excitatory globally inhibitory oscillator network* (LEGION) [42], [50]. They theoretically showed that LEGION can rapidly achieve both synchronization in a locally coupled oscillator group representing each object and desynchronization among a number of oscillator groups representing multiple simultaneously presented objects, thus providing a mechanism for oscillatory correlation. In particular, synchrony in LEGION is achieved at an exponential rate. These features make LEGION different from other oscillator networks which either show indiscriminate synchrony of multiple objects due to long-range connections as pointed out by Sporns *et al.* [41] and Wang [48] or lack an effective mechanism for desynchronization. Motivated by the consideration that time delays are inevitable in signal transmission in both neural and physical systems, Campbell and Wang have recently studied time delays in networks of relaxation oscillators and analyzed the behavior of LEGION with time delays [4]. Their studies show that loosely synchronous solutions can be achieved under a broad range of initial conditions and time delays [4].

We explore both the spiral problem and the inside–outside problem by oscillatory correlation. We attempt to address the following questions. Why humans cannot immediately distinguish between connected and disconnected spirals? What is the connection between the spiral problem and the inside–outside relation? How to perceive the inside–outside relation in neural networks? We show that computation through LEGION with time delays yields a solution to these problems. This investigation leads to the interpretation that perceptual performance would be limited if local activation cannot be rapidly propagated due to time delays. As a special case, we demonstrate that LEGION without time delays reliably distinguishes between connected and disconnected spirals and discriminates the inside and the outside regardless of shape, position, size, and orientation.

The remainder of this paper is organized as follows. Section II overviews the architecture and dynamics of LEGION networks. Section III presents the methodology and reports simulation results. Further discussions are given in Section IV, and conclusions are drawn in Section V.

II. LEGION MODEL

In this section, we first review the architecture and dynamics of LEGION. It is followed by an introduction to a variant with *time delays*. Both of them will be used to deal with the problems in this paper.

As illustrated in Fig. 2(a), the architecture of LEGION is a two-dimensional network (it can be extended to other dimensions). Each oscillator is typically connected to only its neigh-

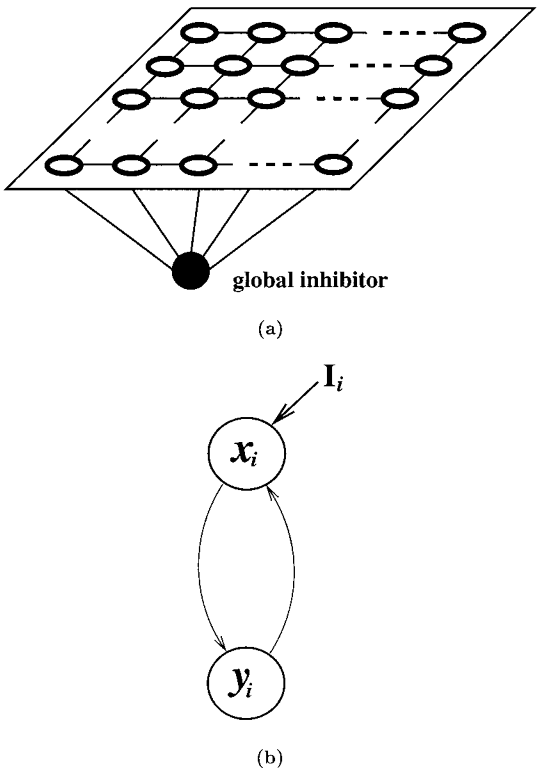


Fig. 2. Architecture of LEGION. (a) Two-dimensional network with four nearest neighbor coupling. The global inhibitor is indicated by the black circle. (b) Structure of a single relaxation oscillator.

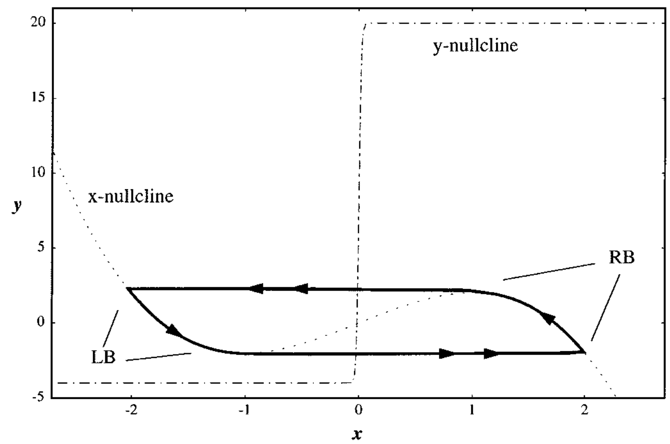


Fig. 3. Nullclines and periodic orbit of a relaxation oscillator as defined in (1) in phase plane. The x-nullcline is shown by the dotted curve, and the y-nullcline is shown by the dash-dot curve. The thick solid curve is the periodic orbit.

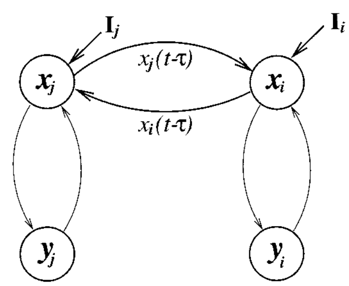


Fig. 4. Two coupled relaxation oscillators with a time delay τ.

bors (four nearest neighbors used in this paper), and the global inhibitor receives excitation from each oscillator on the network and in turn inhibits each oscillator [42], [50]. As depicted in Fig. 2(b), the building block of LEGION, a single oscillator i, is defined as a feedback loop between an excitatory unit x_i and an inhibitory unit y_i [42], [50]

$$\frac{dx_i}{dt} = f(x_i, y_i) + I_i + S_i + \rho \tag{1a}$$

$$\frac{dy_i}{dt} = \epsilon g(x_i, y_i) \tag{1b}$$

where $f(x_i, y_i) = 3x_i - x_i^3 - y_i$, and $g(x_i, y_i) = \lambda + \gamma \tanh(\beta x_i) - y_i$ are used in this paper. I_i represents external stimulation to the oscillator, and S_i represents overall coupling from other oscillators in the network. The symbol ρ denotes the amplitude of a Gaussian noise term which is introduced both to test the robustness of the system and to actively desynchronize different input patterns. The parameter ϵ is chosen to be small, $0 < \epsilon \ll 1$. In this case, (1) without any coupling and noise, corresponds to a standard relaxation oscillator [45]. The x-nullcline of (1a), $dx/dt = 0$, is a cubic function, while the y-nullcline of (1b), $dy/dt = 0$, is a sigmoid function. The parameter β controls the steepness of the sigmoid function and is chosen to be large, $\beta \gg 1$. For an input $I_i > \lambda - \gamma + 2$, the two nullclines intersect only on the middle branch of the cubic, and (1) gives rise to a stable periodic orbit as illustrated in Fig. 3. The periodic solution alternates between a phase of relatively high values of $x(1 \le x \le 2)$ on the right branch (RB) of the cubic, called the *active phase* of the oscillator, and a phase of relatively low values of $x(-2 \le x \le -1)$ on the left branch (LB) called the *silent phase*. Within these two phases, (1) exhibits near steady-state behavior. In contrast, the transition between the phases takes place on a fast time scale; i.e., when the oscillator reaches one extreme point ($x = -1$) of the cubic along LB it will rapidly jump up from LB to RB, and the oscillator will rapidly jump down from RB to LB when it reaches the other extreme point ($x = 1$) along RB. In this case, the oscillator is called *enabled*. For an input $I_i < \lambda - \gamma + 2$, the two nullclines also intersect on the left branch of the cubic, and (1) produces a stable fixed point at a low value of $x(x < 1)$. In this case, the oscillator is called *excitable*. The parameters λ and γ are introduced to control the relative times that an enabled oscillator spends in the two phases, and a larger summation of λ and γ yields a relatively shorter active phase.

In LEGION, the coupling term S_i in (1a) is defined by

$$S_i = \sum_{k \in N(i)} W_{ik} S_\infty(x_k, \theta_x) - W_z S_\infty(z, \theta_z) \tag{2}$$

where

$$S_\infty(x, \theta) = \frac{1}{1 + \exp[-\kappa(x - \theta)]}. \tag{3}$$

W_{ik} is a synaptic weight from oscillator k to oscillator i, and $N(i)$ is the set of adjacent oscillators that connect to i. In this paper, $N(i)$ is four immediate neighbors on a two-dimensional

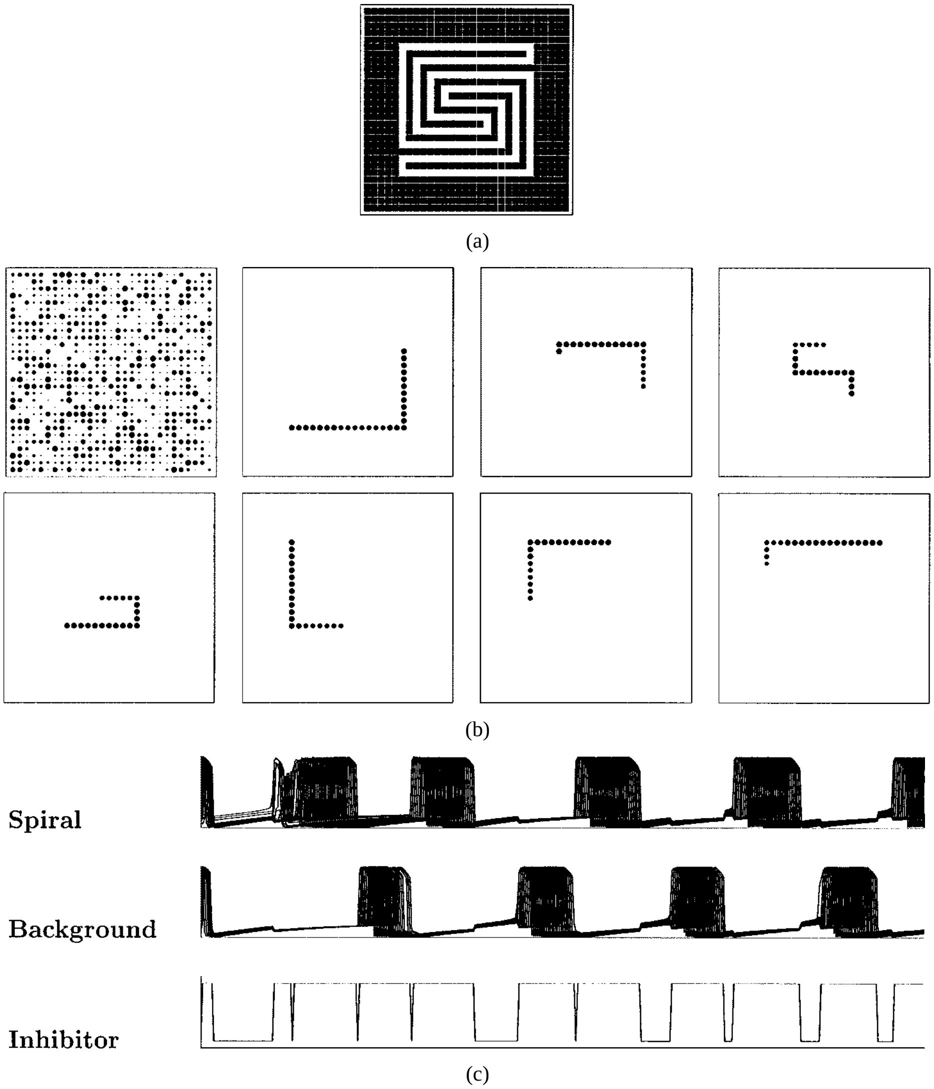


Fig. 5. Results of LEGION with a time delay $\tau = 0.002T$ (T is the period of oscillation) to the spiral problem. The parameter values used in this simulation are $\epsilon = 0.003$, $\beta = 500$, $\gamma = 24.0$, $\lambda = 21.5$, $\alpha_T = 6.0$, $\rho = 0.03$, $\kappa = 500$, $\theta_x = -0.5$, $\theta_z = 0.1$, $\phi = 3.0$, $W_z = 1.5$, $I_s = 1.0$, and $I_u = -1.0$ where I_s and I_u are external input to stimulated and unstimulated oscillators, respectively. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing the spiral, the background, and the global inhibitor, respectively. The simulation took 32 000 integration steps.

TABLE I
MIN–MAX MEASURE FOR THE SPIRAL PROBLEM ($\tau_{\mathrm{RB}} = 74.4$)

measure	Figure 5	Figure 6	Figure 7	Figure 8
T_{max}	283.4	34.9	201.5	23.8
T_{min}	97.4	187.3	101.4	134.5
Min-Max	*no*	*yes*	*no*	*yes*

network, except on the boundaries where $N(i)$ may be either two or three immediate neighbors [no wrap-around, see Fig. 2(a)]. θ_x is a threshold above which an oscillator can affect its neighbors. W_z is a positive weight used for inhibition from the global inhibitor z, whose activity is defined as

$$\frac{dz}{dt} = \phi(\sigma_\infty - z) \tag{4}$$

where $\sigma_\infty = 0$ if $x_i < \theta_z$ for every oscillator, and $\sigma_\infty = 1$ if $x_i \geq \theta_z$ for at least one oscillator. Here θ_z is a threshold. If the activity of every oscillator is below this threshold, then the global inhibitor will not receive any input. In this case, the oscillators on the network will receive no inhibition from the global inhibitor. If, on the other hand, the activity of at least one oscillator is above the threshold, the global inhibitor will receive input. In this case, all of oscillators on the network receive inhibition from the global inhibitor when z is above the threshold θ_z [cf. (2)]. The parameter ϕ determines the rate at which the inhibitor reacts to such stimulation and the parameter κ in (3) is chosen to be large, $\kappa \gg 1$, to produce a steep sigmoid function.

The computation of LEGION can be briefly summarized as follows. Once an oscillator enters the active phase, it triggers the global inhibitor. As a result, the global inhibitor affects the entire network by exerting inhibition on all oscillators according to (2). On the other hand, an active oscillator propagates its activation to its nearest neighbors also according to (2), and the propagation of the activation extends to other oscillators in this manner until all the oscillators representing the same object are activated. Thus, the dynamics underlying LEGION is a process

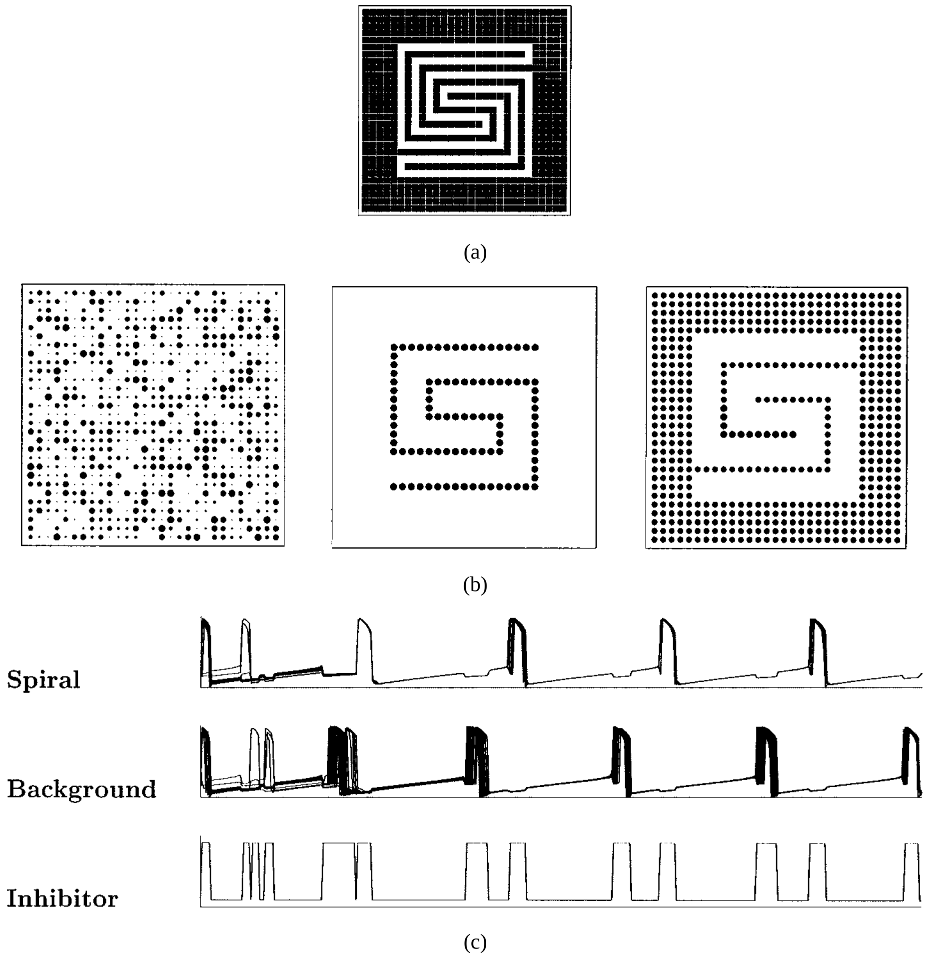


Fig. 6. Results of LEGION without time delays to the spiral problem. The parameter values used in this simulation are the same as listed in the caption of Fig. 5. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing the spiral, the background, and the global inhibitor, respectively. The simulation took 32 000 integration steps.

of both synchronization by local cooperation through excitatory coupling among neighboring oscillators, and desynchronization by global competition via the global inhibitor. A mechanism called dynamic normalization has been suggested to ensure that the quality of synchrony is not influenced by boundary effects [49]. During dynamic normalization weights are updated when a new stimulus pattern is presented and quickly converge to a steady state, in which the total dynamic weights converging to a single oscillator are equal to a constant. Such weight dynamics should be differentiated from neural network learning, where training patterns are repeatedly presented. In the simulations reported in the paper, such a mechanism is adopted.

Recently, Campbell and Wang studied time delays in networks of relaxation oscillators and analyzed LEGION networks with time delays [4]. Their investigation consists of formal analysis of two coupled oscillators and empirical study of networks of relaxation oscillators. Due to time delays, perfect synchrony may not be achieved for coupled oscillators. Campbell and Wang introduced a concept called *loose synchrony* to describe the behavior of relaxation oscillators with time delays [4]; that is, two oscillators are loosely synchronous if the phase difference between them is less than or equal to the time delay. Based on the definition, Campbell and Wang analytically showed that when a time delay is less than a specific upper bound, τ_{RM}, loose synchrony of two oscillators is achieved within a considerable range of initial time differences and coupling strengths [4]. Here $\tau_{\rm RM} = \min\{\tau_{\rm LB}, \tau_{\rm RB}\}$, where $\tau_{\rm LB}$ and $\tau_{\rm RB}$ are the times that an oscillator spends to traverse LB and RB (see Fig. 3), respectively. They also showed that for $\tau > \tau_{\rm RM}$, loose synchrony is rarely achieved. For locally coupled networks of relaxation oscillators, their empirical investigation indicates that loosely synchronous solutions are attained under similar conditions [4].

As illustrated in Fig. 4, time delays affect the coupling of coupled oscillators, but the architecture of LEGION remains unchanged though the behavior of LEGION with time delays can be different [4]. Equations (1)–(4), therefore, can be still used with a modified coupling term [(2)] as follows:

$$S_i = \sum_{k \in N(i)} W_{ik} S_\infty(x_k(t-\tau), \theta_x) - W_z S_\infty(z, \theta_z) \quad (5)$$

where τ ($\tau > 0$) is a time delay in lateral interactions and other parameters are the same as defined in (2).

III. Simulation Results

In this section, we first present our methodology for simulations. Then, we report simulation results in details. Some analysis for simulation results is also given.

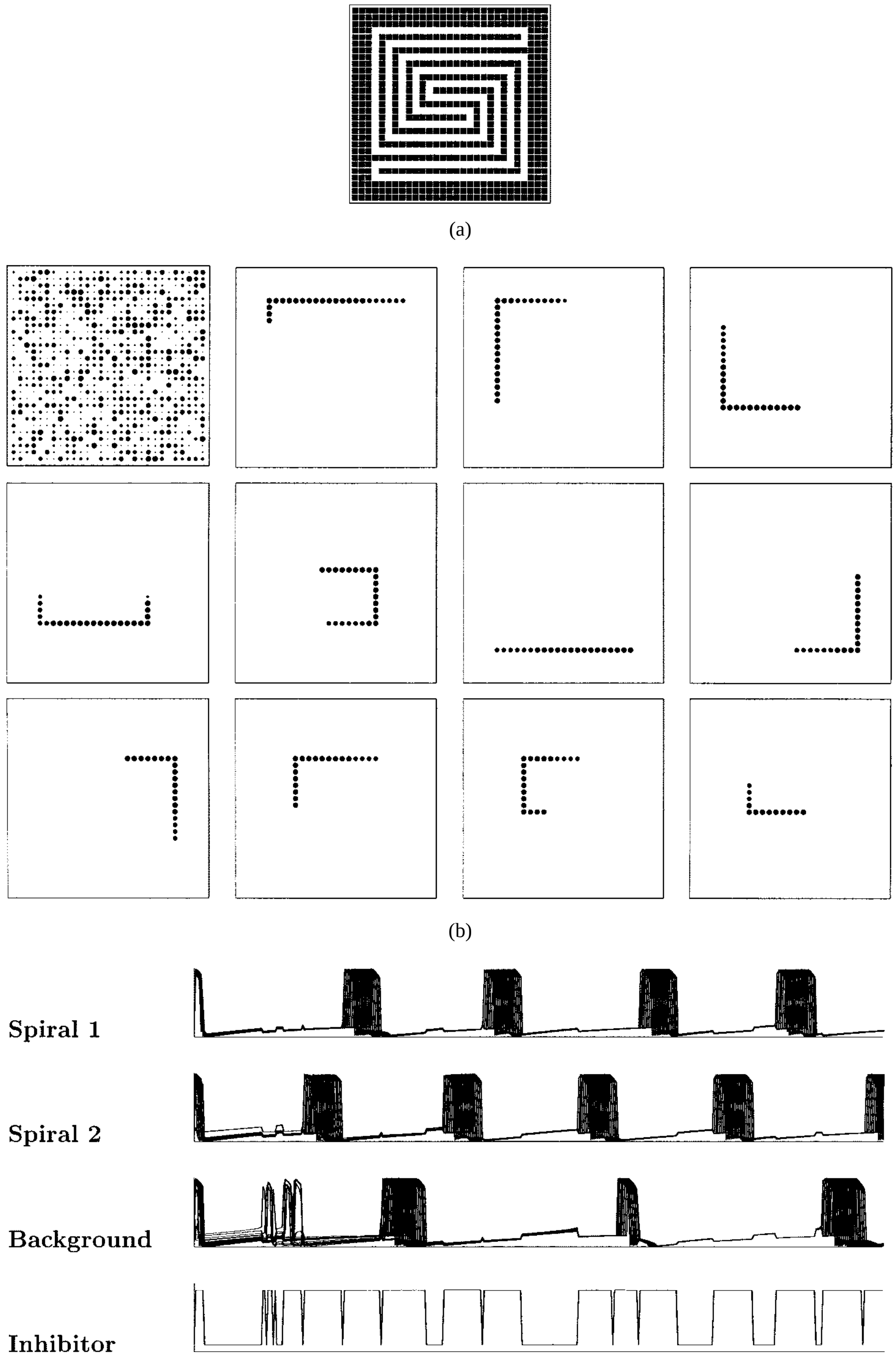


Fig. 7. Results of LEGION with a time delay $\tau = 0.002T$ to the spiral problem. The parameter values used in this simulation are the same as listed in the caption of Fig. 5. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing the double spirals, the background, and the global inhibitor, respectively. The simulation took 36 000 integration steps.

A. Methodology

For a given image consisting of $N \times N$ pixels, a two-dimensional LEGION network with $N \times N$ oscillators is employed so that each oscillator in the network corresponds to a pixel in the image. In this paper, all images are binary to be consistent with typical stimuli used in the original problems. In converting outline patterns such as Fig. 1 to an image representation using binary pixels, we adopt the following convention. Black pixels correspond to either figure or background (both are white in outline patterns), and white pixels correspond to outlines (boundaries separating figures and the background). This convention facilitates presentations on white paper, and does not affect the system outcome. For filled-in patterns on a white background such as Fig. 9, however, black pixels correspond to figures and white pixels correspond to the background. In this case, unlike outline patterns, a human subject can readily realize and ignore

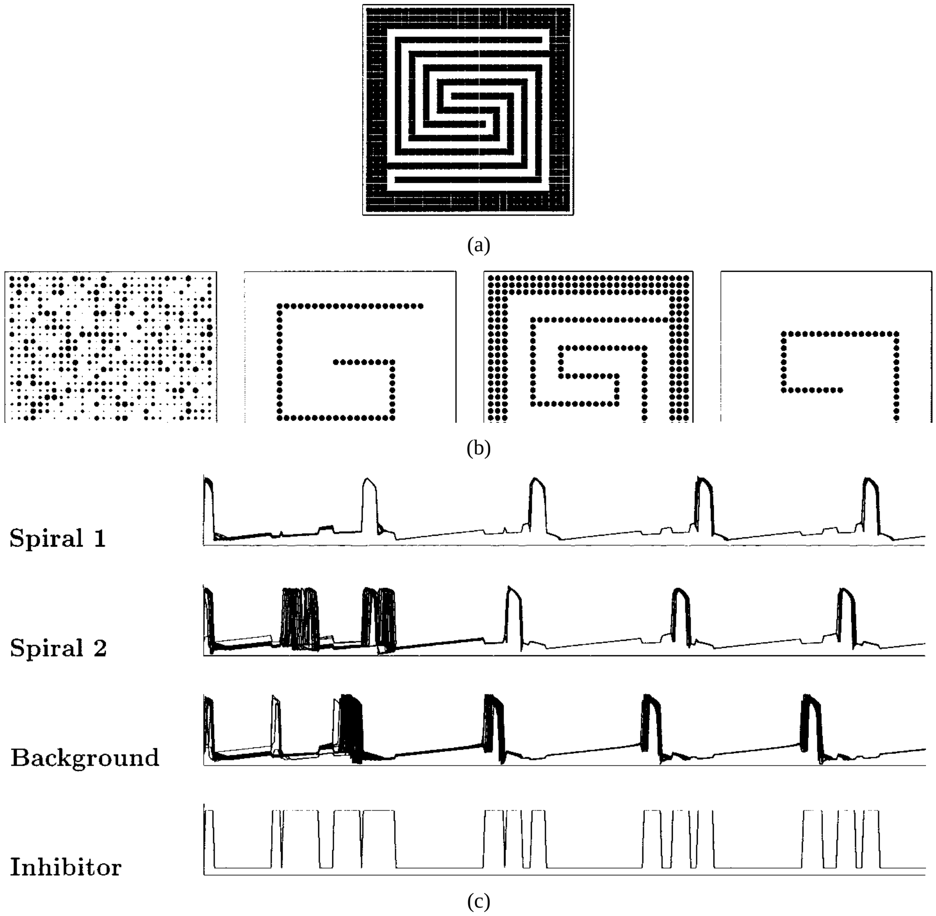


Fig. 8. Results of LEGION without time delays to the spiral problem. The parameter values used in this simulation are the same as listed in the caption of Fig. 5. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing the double spirals, the background, and the global inhibitor, respectively. The simulation took 36 000 integration steps.

the (white) background. In simulations, we let the external input of an oscillator corresponding to a black pixel be positive, and the external input corresponding to a white pixel be negative.

An effective connection refers to the connection between two oscillators through which they can directly affect each other. Since each oscillator in LEGION is connected to only its four nearest neighbors, an effective connection can be only established if and only if two oscillators are adjacent and both of them are activated by external stimulation; otherwise, there is no effective connection between them. Suppose that an oscillator, O_i, has K_i neighbors with positive input and α_T is a constant used for dynamic normalization. Then, the effective strength, W_{ij}, between any two oscillators O_i and O_j is dynamically determined as

$$W_{ij} = \begin{cases} \alpha_T/K_i & \text{if } O_j \text{ is a neighbor of } O_i \\ & \quad \text{and external input to both is positive} \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Note that due to this dynamic normalization [49], the overall sum of coupling strengths of any stimulated oscillator with at least one effective connection is α_T. Other parameters in LEGION can be chosen according to the theoretical analysis of Terman and Wang [42].

We use pattern formation in oscillator networks to refer to the behavior that all the oscillators representing the same object are synchronous, while the oscillators representing different objects are desynchronous. Terman and Wang have analytically shown that pattern formation can be achieved in LEGION without time delays [42]. However, it may not be achieved when time delays are introduced. Although the loose synchrony concept has been introduced to describe time delay behavior [4], it does not indicate pattern formation in an entire network even when loose synchrony is achieved. This is because loose synchrony is a local concept defined in terms of pairs of neighboring oscillators.

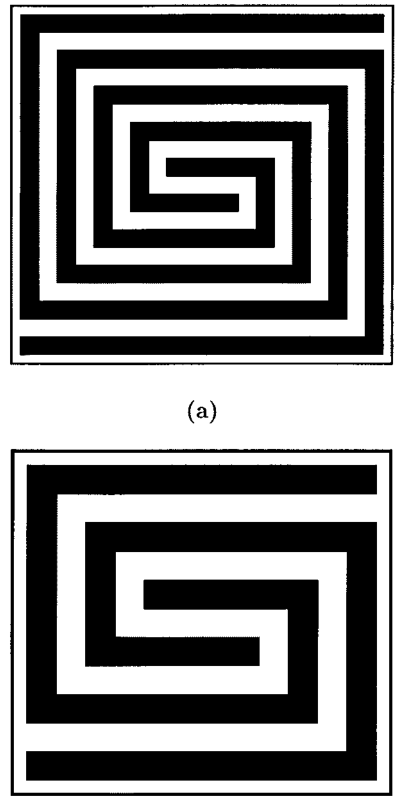
Here we introduce a measure called *min–max difference* in order to examine whether pattern formation is achieved. Suppose that oscillators O_i and O_j represent two pixels in the same object, and the oscillator O_k represents a pixel in a different object. Moreover, let t^s denote the time at which oscillator O_s enters the active phase. The min–max difference measure is defined as

$$T_{\max} < \tau_{\mathrm{RB}} \quad \text{and} \quad T_{\min} \geq \tau_{\mathrm{RB}} \tag{7}$$

where $T_{\max} = \max_{O_i,\, O_j \text{ in same object}} |t^i - t^j|$ and $T_{\min} = \min_{O_i,\, O_k \text{ in different objects}} |t^i - t^k|$. In other words, the minimum difference in jumping times between any two oscillators representing different objects is greater than or equal to an active phase, whereas the maximum difference in jumping times (of the same period) between any two oscillators representing the same object is less than an active phase. When (7) is satisfied, we say that pattern formation takes place. Intuitively, this measure suggests that pattern formation is achieved if any two oscillators representing two pixels in the same object have some overlap in the active phase, while any two oscillators representing two pixels belonging to different objects never stay in the active phase simultaneously. As a result, pattern formation is achieved by a LEGION network within $P \times T$ where T is the period of oscillation which can be calculated explicitly in terms of system parameters (see the Appendix) and P is the number of objects in the presented image. This statement is derived by Terman and Wang [42].

We adopt two methods to display our simulation results. One is the combined x activities of the oscillators representing the same object as well as the global inhibitor over time, and the other is a set of snapshots to show instantaneous activity of the entire network at various stages of dynamic evolution. For snapshots, we use black circles to show x activities of oscillators and stipulate that the diameter of a black circle is proportional to $(x - x_{\min})/(x_{\max} - x_{\min})$, where x values of all oscillators lie in the interval $[x_{\min}, x_{\max}]$.

It is well known that information transmission in a biological system is inevitably associated with time delays which may alter the behavior of the system. Thus, it is desirable to use LEGION with time delays. The performance of LEGION without time delays is shown as a special case to illustrate its capabilities of solving problems.



(a)

(b)

Fig. 9. Two-spiral problem. (a) Two spirals with many turns. (b) Two spirals with few turns.

B. Results

In the following simulations, the differential equations in (1)–(5) are numerically solved using the fourth-order Runge–Kutta method with a step size 0.2 (smaller steps were used to verify the results). We illustrate stimulated oscillators with black squares. All oscillators are initialized randomly. A large number of simulations have been conducted, and similar results as reported here are produced with a broad range of parameter values and network sizes. Here we report typical results using a specific set of parameter values.

1) Spiral Problem: As illustrated in Fig. 1, the spiral problem consists of two images proposed by Minsky and Papert [29], [30]. In Fig. 1(a), it is a connected single spiral, while Fig. 1(b) contains disconnected double spirals. For simulations, they are sampled as two binary images with 29×29 pixels. For these images, two questions can be addressed: 1) When an image is presented, can one determine whether it contains a single spiral or double spirals? 2) Given a point on a two-dimensional plane, can one determine whether it is inside or outside a specific spiral?

We first apply LEGION with time delays to the single spiral image. Fig. 5(a) illustrates the visual stimulus, where black pixels correspond to stimulated oscillators and white ones unstimulated oscillators. Fig. 5(b) shows a sequence of snapshots after the network is stabilized except for the first snapshot which shows the random initial state of the network. These snapshots are arranged in temporal order first from left to right and then from top to bottom. Fig. 5(c) shows the temporal trajectories of the combined x activities of the oscillators representing the spiral and the background as well as the temporal activity of the global inhibitor. We observe from these snapshots that an active oscillator in the spiral propagates its activation to its two immediate neighbors with some time delay, and the process of propagation forms a traveling wave along the spiral. We emphasize that, at any time, only the oscillators corresponding to a portion of the spiral stay in the active phase together, and the entire spiral can never be in the active phase simultaneously. Pattern formation is examined based on the min–max measure. In Table I, $T_{\max}$ and $T_{\min}$ [see (7)] are given, together with the value of τ_{RB}. In this case, pattern formation is not achieved because $T_{\max} > \tau_{\mathrm{RB}}$. Thus, based on the oscillatory correlation theory, our system cannot group the whole spiral together. The system's limitation is qualitatively consistent with that of human performance in this situation (see Section I). Also, the process that our model traces the spiral bears resemblance to the way humans sequentially trace the spiral for identification.[1] Note that the convoluted part of the background behaves similarly.

To illustrate the effects of time delays, we apply LEGION without time delays to the same image, and Fig. 6 shows the results. Fig. 6(a) illustrates the same visual stimulus. Fig. 6(b) shows three snapshots corresponding to the random initial state of the network, the separated spiral and the background, respectively. Fig. 6(c) illustrates the temporal trajectories of the stim-

[1]We recognize that eye movements may be involved in tracing convoluted spirals by humans. Eye movements are usually preceded by covert shift of attention [37]. Thus, a more rigorous treatment of sequential tracing likely involves an interplay between covert attentional shifts, whose time scales are roughly comparable with those of neural oscillations, and eye movements (or overt shifts), whose time scales are substantially slower.

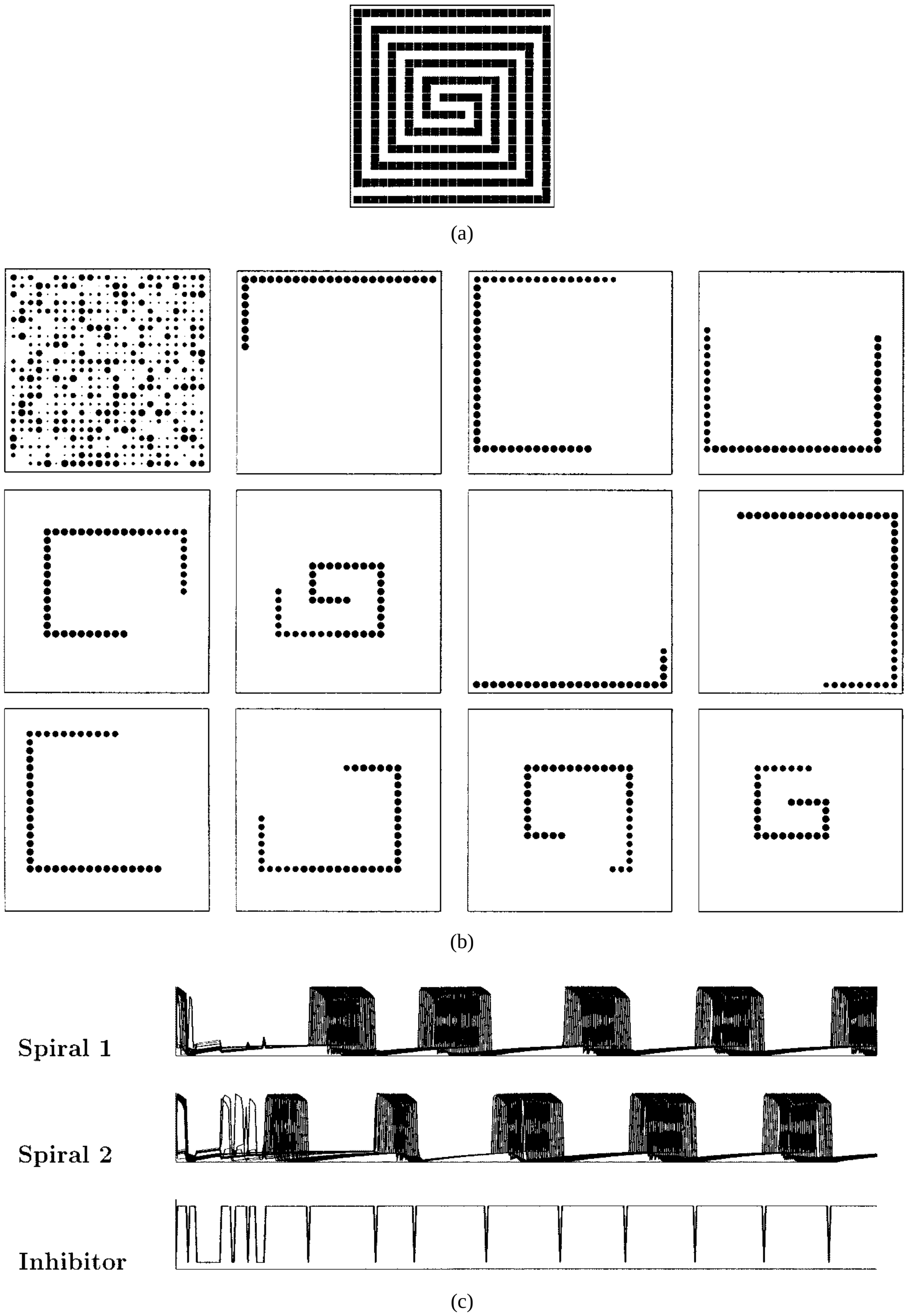
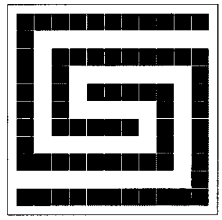


Fig. 10. Results of LEGION with a time delay $\tau = 0.002T$ to the two-spiral problem of Fig. 9(a). The parameter values used in this simulation are $\epsilon = 0.004$, $\gamma = 14.0$, $\lambda = 11.5$ and the other parameter values are the same as listed in the caption of Fig. 5. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing the two spirals and the global inhibitor, respectively. The simulation took 24 000 integration steps.

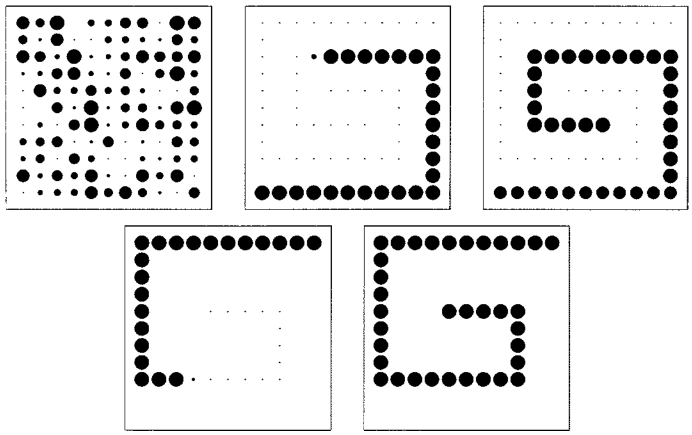
ulated oscillators. From Table I, it is clear that pattern formation is achieved, and the single spiral can be segregated from the background by the second period. Thus, LEGION without time delays can readily solve the spiral problem in this case. The failure to group the spiral in Fig. 5 is caused by time delays in the coupling of neighboring oscillators.

We also apply LEGION with time delays to the double spirals image. Fig. 7(a) shows the visual stimulus. Fig. 7(b) shows a sequence of snapshots. Similar behavior as illustrated in Fig. 5 occurs here. The results show that the pixels in any one of double spirals cannot be grouped to the same pattern. The behavior of the system for the convoluted part of the background is similar to that for the double spirals. Fig. 7(c) illustrates the temporal trajectories. Our observation beyond the duration of Fig. 7(c) shows that the system is stabilized by the fourth period of oscillations. As indicated by Table I, pattern formation is not achieved after the network is stabilized.

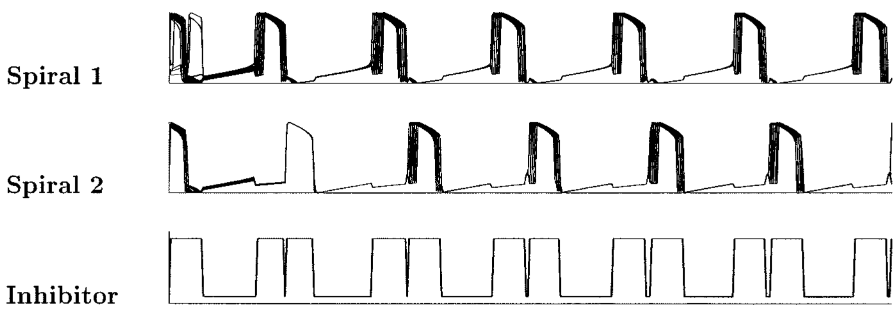
We also apply LEGION without time delays to the double spirals image for the same purpose as described before, and Fig. 8 shows the results in the same manner as in Fig. 6. As shown in Table I, pattern formation is achieved, and one spiral can be segregated from both the other spiral and the background by



(a)



(b)



(c)

Fig. 11. Results of LEGION with a time delay $\tau = 0.002T$ to the two-spiral problem of 9(b). The parameter values used in this simulation are the same as listed in the caption of Fig. 10. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing the two spirals and the global inhibitor, respectively. The simulation took 24 000 integration steps.

TABLE II
MIN–MAX MEASURE FOR THE TWO-SPIRAL PROBLEM ($\tau_{\mathrm{RB}} = 106.0$)

measure	Figure 10	Figure 11	Figure 12	Figure 13
T_{max}	318.5	18.7	12.5	15.8
T_{min}	112.8	132.6	242.8	234.5
Min-Max	*no*	*yes*	*yes*	*yes*

the second period. Thus, LEGION without time delays readily solves the spiral problem in this case, and it indicates that the failure to group the double spirals in Fig. 7 results from time delays.

For the spiral problem, pattern formation means that one can provide the answers to the aforementioned questions of counting the number of objects or identifying whether two pixels belong to the same spiral or not. No answer is available when pattern formation is not achieved. Hence, our system cannot solve the spiral problem in general. Only under the special condition of no time delay can the system solve the problem.

2) Two-Spiral Problem: A variant of the spiral problem, called *two-spiral* problem, is well known as a benchmark task in neural networks [10]. Fig. 9 illustrates two cases of the two spirals with different complexities; i.e., many turns in Fig. 9(a) and few turns in Fig. 9(b). The two-spiral problem may be viewed as a modified version of the double spirals image in Fig. 1(b), where two spirals are distinguished from the background through the use of color. The benchmark problem is described as follows: given a point in one of two spirals, identify which spiral it belongs to? The problem may be converted into a problem of perceiving the inside–outside relation: whether a point is inside a specific spiral? The human performance to the problem is that perceiving the inside–outside relation is effortless when spirals have few turns, but effortful when spirals have many turns [46], [44].

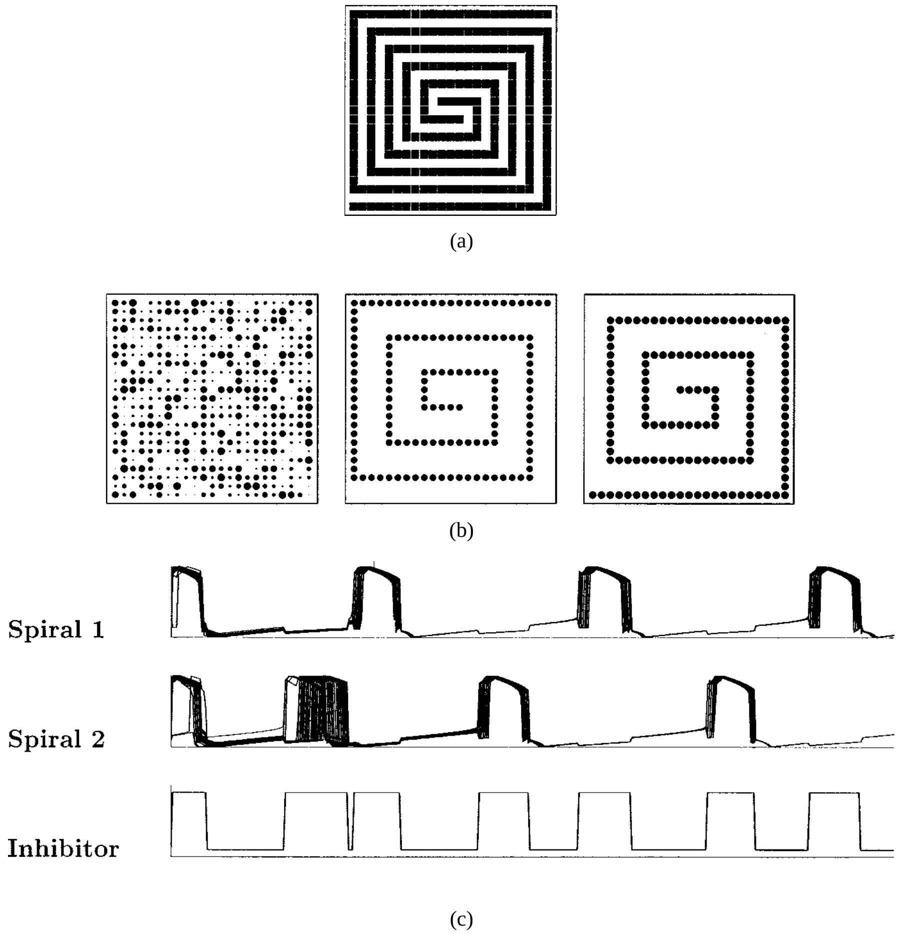


Fig. 12. Results of LEGION without time delays to the two-spiral problem of Fig. 9(a). The parameter values used in this simulation are the same as listed in the caption of Fig. 10. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing the two spirals and the global inhibitor, respectively. The simulation took 12 000 integration steps.

For simulations, pictures in Fig. 9 are sampled as binary images with 23×23 and 11×11 pixels, respectively. LEGION with time delays is first applied to both images, and Figs. 10 and 11 show the results, respectively. Figs. 10(a) and 11(a) illustrate the visual stimuli, where black pixels represent two spirals and white pixels represent the background. Note that this convention is somewhat different from Section III-B1 (see explanation in Section III-A). Figs. 10(b) and 11(b) show a sequence of snapshots after the networks are stabilized, except for the first snapshot which shows the random initial states of networks. Figs. 10(c) and 11(c) show the temporal trajectories. As indicated in Table II, pattern formation cannot be achieved for the case in Fig. 9(a), but is achieved for the case in Fig. 9(b).

We observe from those snapshots that, like its behavior for the spiral problem, an active oscillator in a spiral only propagates its activation to its immediate neighbors with some time delay, and the process of propagation forms a traveling wave along each spiral. The snapshots in Fig. 10(b) reveal that at any time only the oscillators corresponding to a portion of a specific spiral stay in the active phase together, and the entire spiral is never in the active phase at the same time. In contrast, however, Fig. 11(b) reveals that all the oscillators corresponding to a specific spiral can stay in the active phase simultaneously though they may enter the active phase at different times in the same period. That is, the system can group a whole spiral together when there are few turns in the spiral. However, it fails to group a whole spiral when there are more turns in the spiral. This is because the more turns in a spiral lead to the longer propagation time along the spiral, and there is no sufficient time to allow all the oscillators representing the spiral to stay in the active phase simultaneously. This limitation of our system qualitatively matches that of human performance.

To illustrate the effects of time delays, we apply LEGION without time delays to the two images in Fig. 9, and Figs. 12 and 13 show the corresponding results. Pattern formation is achieved in both cases regardless of the spiral complexity (see Table II), and the two spirals are segregated in both cases by the second period.

3) Inside–Outside Relations: The inside–outside problem refers to whether a dot belongs to area A or to area B, as illustrated in Fig. 14. The perception of inside–outside relations is usually performed by humans with intriguing efficiency. Fig. 14(a) is such an example, where humans can effortlessly recognize the dot is inside area A rather than area B though the two areas are separated by a convoluted boundary. However, immediate perception is subject to limitations. For instance, humans cannot immediately identify whether the dot is inside area A or area B for the visual stimulus in Fig. 14(b). For simulations, the two pictures are sampled as binary images with 43×43 pixels. Here, we report our typical results.

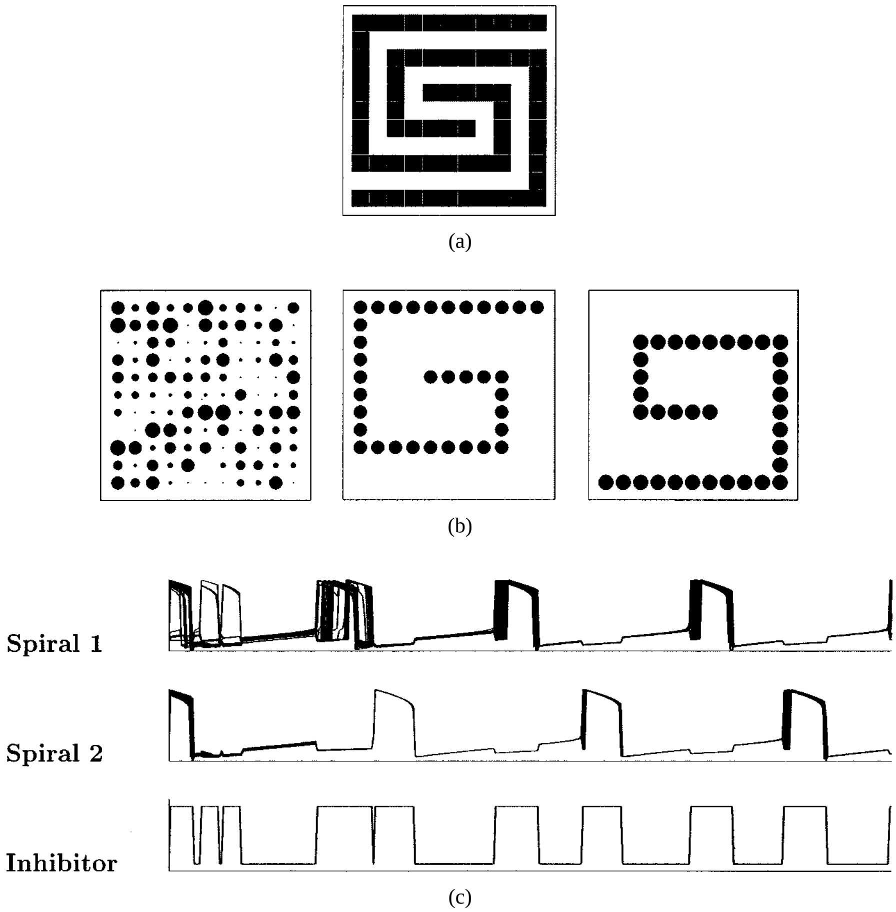


Fig. 13. Results of LEGION without time delays to the two-spiral problem in Fig. 9(b). The parameter values used in this simulation are the same as listed in the caption of Fig. 10. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing the two spirals and the global inhibitor, respectively. The simulation took 12 000 integration steps.

We first apply LEGION with time delays to the two images in Fig. 14, and Figs. 15 and 16 show the results. Note that the visual stimuli here are represented following the same convention as in Section III-B1. We observe from Fig. 15(b) that the activation of an oscillator can rapidly propagate through its neighbors to other oscillators representing the same area, and eventually all the oscillators representing the same area (A or B) stay together in the active phase, though they generally enter the active phase at different times due to time delays. As shown in Table III, pattern formation is achieved, and area A is separated from area B by the second period. In contrast, we observe from Fig. 16(b) that although an active oscillator rapidly propagates its activation in open regions as shown in the last three snapshots, propagation is limited once the traveling wave propagates in spiral-like regions as shown in earlier snapshots. This is because it takes longer or much longer to travel a convoluted region from one end to another. As a result, the oscillators representing the whole area are never in the active phase simultaneously, and pattern formation is not achieved after the network is stabilized (see Table III). Thus, our system cannot group the whole area and fails to identify the pixels of one area as belonging to the same pattern. Again, this limitation is qualitatively consistent with that of human performance in this situation (see Section I).

We also apply LEGION without time delays to the two images in Fig. 14 (see Section I). Figs. 17 and 18 show the results. These figures show that LEGION without time delays readily segregates two areas, and pattern formation is achieved in both cases (see Table III). Area A is separated from area B by the second period in both cases. Thus, the failure to group each area in Fig. 16 is attributed to time delays in the coupling.

In general, the above simulations suggest that oscillatory correlation provides a way to address inside–outside relations by a neural network; when pattern formation is achieved, a single area segregates from other areas that appear in the same image. For a specific dot on the two-dimensional plane, the inside–outside relations can be identified by examining whether the oscillator representing the dot synchronizes with the oscillators representing a specific area.

4) Role of Time Delay: As described in Section II, the LEGION model with the time delay τ was first described by Campbell and Wang [4], who carried out a comprehensive analysis of such a system. In terms of oscillatory dynamics, this paper is a direct application of their analysis. The interested reader is referred to their paper for an extensive treatment. Relating to the present study, we point out that the same value of τ, which equals 0.2% of the period of oscillation, is used in all the above simulations. The choice of this relatively small τ is justified by the fact that only adjacent oscillators are coupled. As we mentioned earlier, similar results were obtained in our simulations when τ is varied within a reasonable range. Fixing other system parameters, increasing τ prolongs the traveling

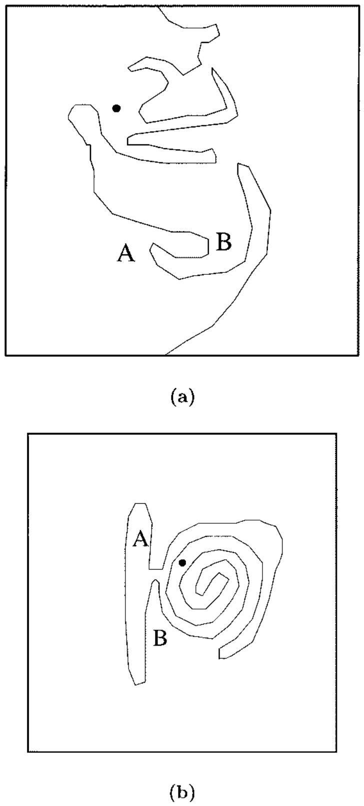


Fig. 14. Inside–outside relations. (a) An example where immediate perception is available (adapted from Julesz [22]). (b) An example where immediate perception is not available (adapted from Ullman [43]).

wave going through the same input pattern [see, for example, Fig. 5(c)], thus increasing the likelihood that the pattern will not be grouped together. In other words, the pattern is less likely to be perceived as a whole at a time. Similarly, when τ is fixed, the more convoluted an input pattern is, the longer the system takes to propagate a traveling wave through the entire pattern and thus the pattern is less likely to be perceived as a whole.

IV. Discussion

The relaxation oscillator, the building block of LEGION, is dynamically very similar to numerous other oscillators used in modeling neuronal behavior, such as the FitzHugh–Nagumo model [12], [33], and the Morris–Lecar model [31]. These models can all be viewed as simplifications of the Hodgkin–Huxley equations of neuronal activity. Recently, Campbell showed that equations [such as (1)] defining relaxation oscillations can generate in different parameter regimes different classes of oscillations, including harmonic (sinusoidal) and integrate-and-fire oscillations, and thus they are of generic nature [3]. Oscillatory correlation is consistent with growing evidence that supports the existence of neural oscillation in the visual cortex and other brain regions [40]. Furthermore, the local excitatory connections adopted in the network are motivated by various lateral connections in the brain, in particular, the horizontal connections in the visual cortex [14], [15]. The global inhibitor on the other hand, might be viewed as a neuronal group in the thalamus, and its activity may be interpreted as the collective behavior of the neuronal group. On the other hand, recent theoretical studies on perceptual organization have shown that the formation of uniform and connected regions is probably the first step in perceptual organization [36]. This suggestion agrees with the emphasis on emergent synchrony based on local connectivity, which reflects both connectedness and uniformity. The binding mechanism in LEGION provides a neurocomputational foundation to generate uniform and connected regions.

Physiological studies also show that there are waves of neural activity traveling with velocities of 10–90 cm/s in the hippocampus and other cortical areas [26], [28]. One mechanism of generating traveling waves is time delays caused by nerve conduction [28]. In local cortical circuits, for instance, the speed of nerve conduction is less than 1 m/s [32] such that connected neurons with 1 mm apart have a time delay of more than 4% of the period of oscillation assuming 40 Hz oscillations. Therefore, the use of LEGION with time delays represents a general case, while the use of LEGION without time delays should be regarded as a special case.

From simulations, we have observed that it is often difficult for LEGION with time delays to group a long and narrow region such as a spiral. For such visual stimuli, our network can achieve pattern formation only if a spiral-like structure has relatively few turns. Our simulation results can be interpreted as follows. In a locally coupled neural network, an active oscillator can only propagate its activation to its neighbors. Thus, the spiral-like regions constrain propagation of the local activation so that activation slowly spreads along the spiral structure. Such propagation may take too long for the oscillators representing the spiral-like region to stay in the active phase simultaneously. On the other hand, all the problems studied in this paper are associated with making a decision on *connectedness*, i.e., given any two points whether or not there is a path in the figure through adjacent points that connects the two points [29], [30]. To make a correct decision, global information is necessary. However, global information is not available until pattern formation is achieved in the framework of oscillatory correlation. The dilemma between local activation propagation and need of global information might be responsible for human limitations in perceiving such visual stimuli.

With additional information discriminating spirals and background, humans can solve the spiral problem efficiently. Fig. 19 illustrates such an example where two spirals and the background can be distinguished on the basis of texture. How is such facilitation obtained? We explain this facilitation in terms of long-range connections that may be established between oscillators. In our simulations, only nearest neighbor connections are employed. With longer connections, we expect that synchronization between remote oscillators is facilitated and thus grouping between pixels corresponding to the same structure is improved. In fact, there exist long-range lateral connections among neurons in the visual cortex and other brain regions. However, such long-range connection would need to be constrained in perceiving spiral-like structures as in the original form of Fig. 9, because pixels constituting figure and background are identical and both figure and background are narrow structures. In this situation long-range connections would confuse figure and background. This is part of the justification for

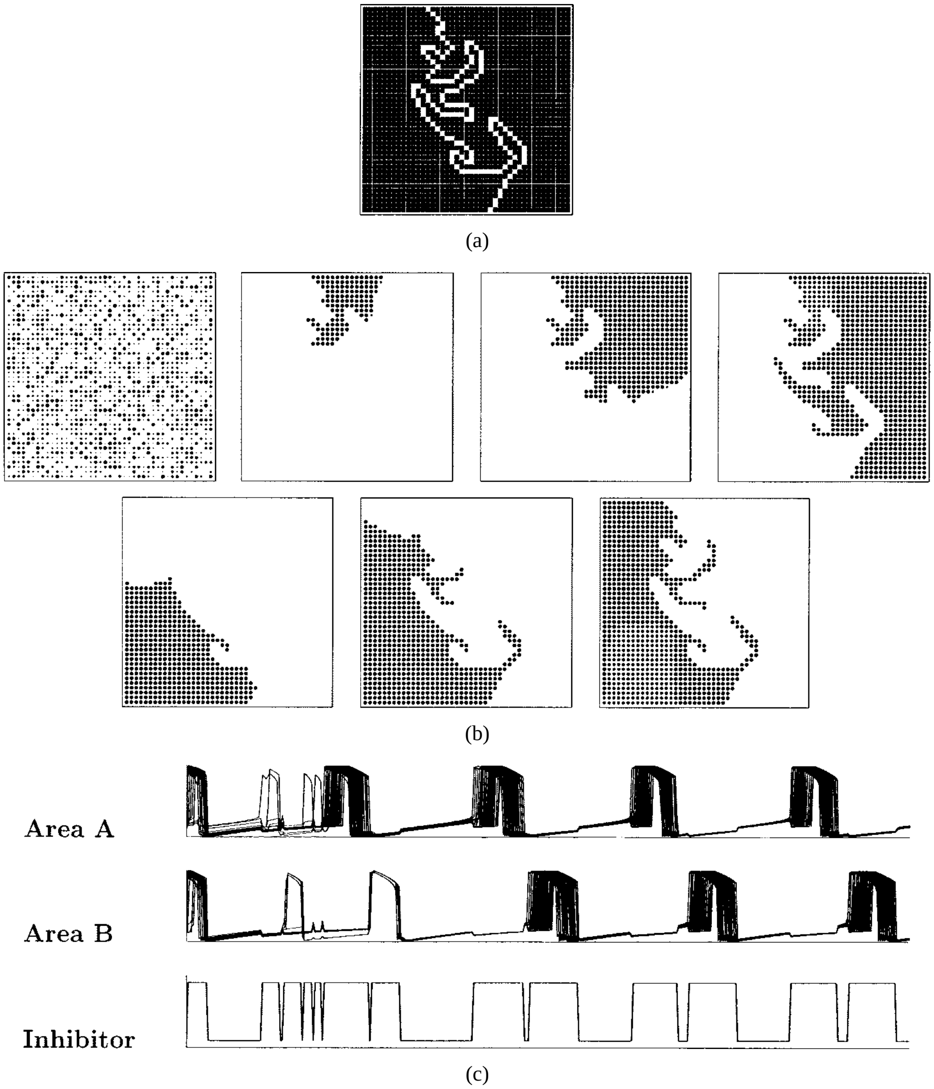


Fig. 15. Results of LEGION with a time delay $\tau = 0.002T$ to the inside–outside relation of Fig. 14(a). The parameter values used in this simulation are the same as listed in the caption of Fig. 10. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing areas A and B as well as the global inhibitor, respectively. The simulation took 24 000 integration steps.

our use of nearest neighbor coupling, which is a gross simplification of local cortical circuitry. On the other hand, information such as texture, motion, color can facilitate the effective use of long-range connections. With long-range connections, we expect that our network with time delays can solve the type of the spiral problem exemplified in Fig. 19. In light of the above discussions, it is clear that, besides the length of a spiral, the difficulty of telling the spiral apart is determined by how convoluted (such as the number of turns as illustrated in Fig. 9) its boundary is and how easily it distinguishes from the rest of the figure in terms of color, texture, etc. These factors affect the extent of lateral connections, and thus the range of synchrony in the network. On the other hand, the use of longer connections in a network would introduce new issues, such as what appropriate connection scales should be used and how it can be done automatically without rewiring. Another issue concerns size invariance. As mentioned in Section I, the time for humans to perform curve tracing is, within a considerable range, invariant to the size of the input pattern. This calls for a mechanism that handles size variations before our network is applied. These issues are left to future research for a more comprehensive treatment of the spiral problem under various variations.

As described earlier, as a benchmark task the two-spiral problem is well known in the neural-network community. Although it has been reported that many neural network models can solve the problem through learning, their solutions are limited because generalization abilities of resulting learning systems highly depend on the training set. When slightly different spirals (e.g., shape, position, size, or orientation) are used as input to learned systems, the correct solution cannot be generated. As pointed out by Minsky and Papert [29], [30], solving the spiral problem is equivalent to detecting connectedness, and connectedness cannot be computed by any diameter-limited or order-limited perceptrons. This limitation holds for multilayer perceptrons regardless of learning scheme [30, p. 252]. Unfortunately, few authors have discussed generality of their solutions. In contrast, LEGION provides a generic solution to the spiral problem. Our simulations have shown that LEGION without time delays can always distinguish these figures regardless of shape, position, size, and

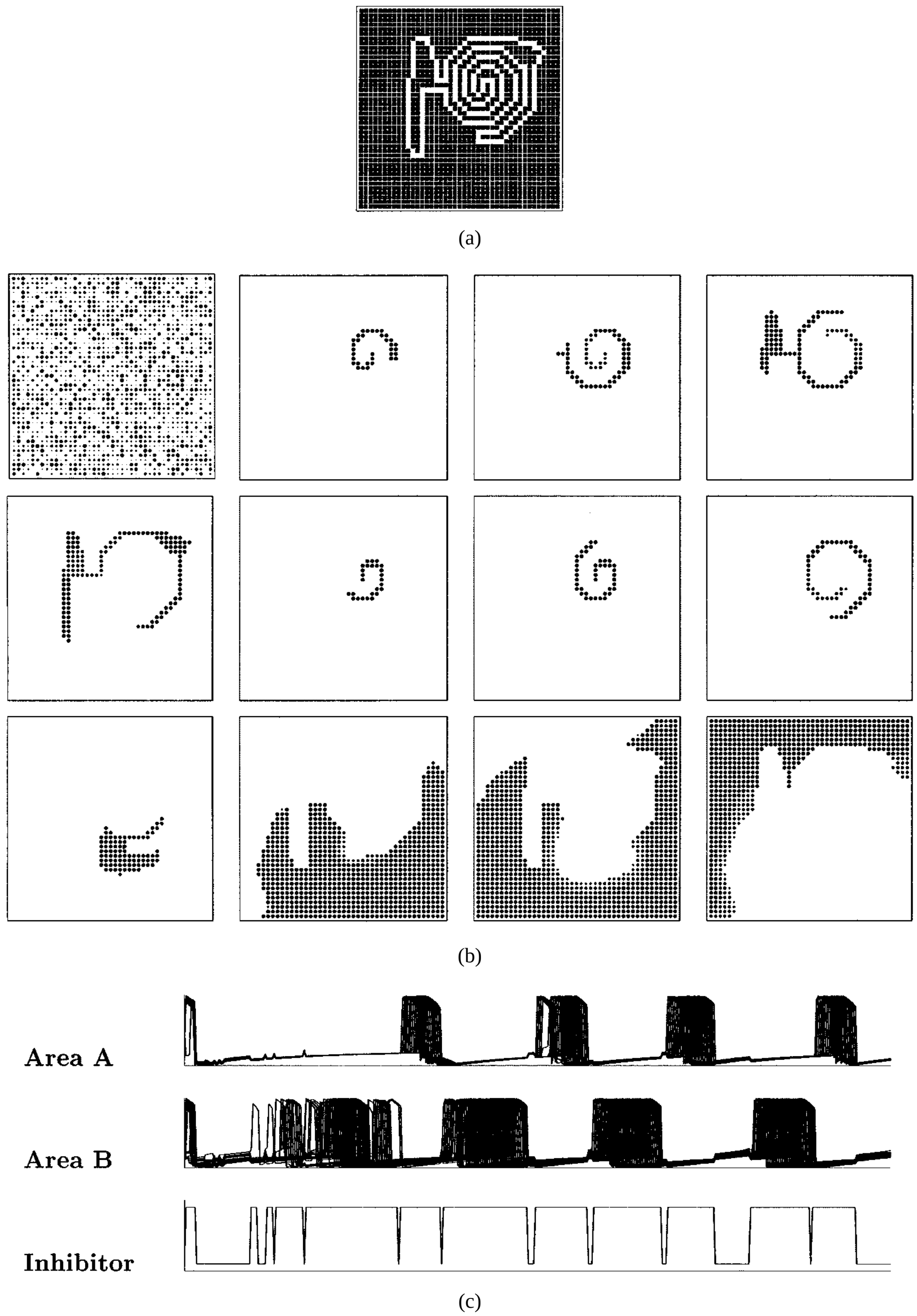


Fig. 16. Results of LEGION with a time delay $\tau = 0.002T$ to the inside–outside relation of Fig. 14(b). The parameter values used in this simulation are the same as listed in the caption of Fig. 10. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing areas A and B as well as the global inhibitor, respectively. The simulation took 24 000 integration steps.

TABLE III
MIN–MAX MEASURE FOR THE INSIDE/OUTSIDE PROBLEM ($\tau_{\mathrm{RB}} = 106.0$)

measure	Figure 15	Figure 16	Figure 17	Figure 18
T_{max}	78.2	328.4	32.4	35.8
T_{min}	142.6	119.7	167.5	228.2
Min-Max	*yes*	*no*	*yes*	*yes*

orientation. In particular, no learning is involved in LEGION. Therefore, we suggest that this kind of geometric problems may be better solved by a properly connected architecture with the oscillatory correlation representation than by supervised learning from examples. The latter approach, when dealing with geometric patterns, faces the challenge that there potentially are exponentially many instances of a geometric pattern (e.g., connectedness), too many to be feasible for generalization from a limited set of training examples (see also [30]). By this suggestion, we do not mean to downplay the importance of supervised learning; rather, we emphasize that architecture and representation, along with learning, are all important for neural computation.

According to Ullman [43], [44], the perception of inside–outside relation involves a complicated procedure of

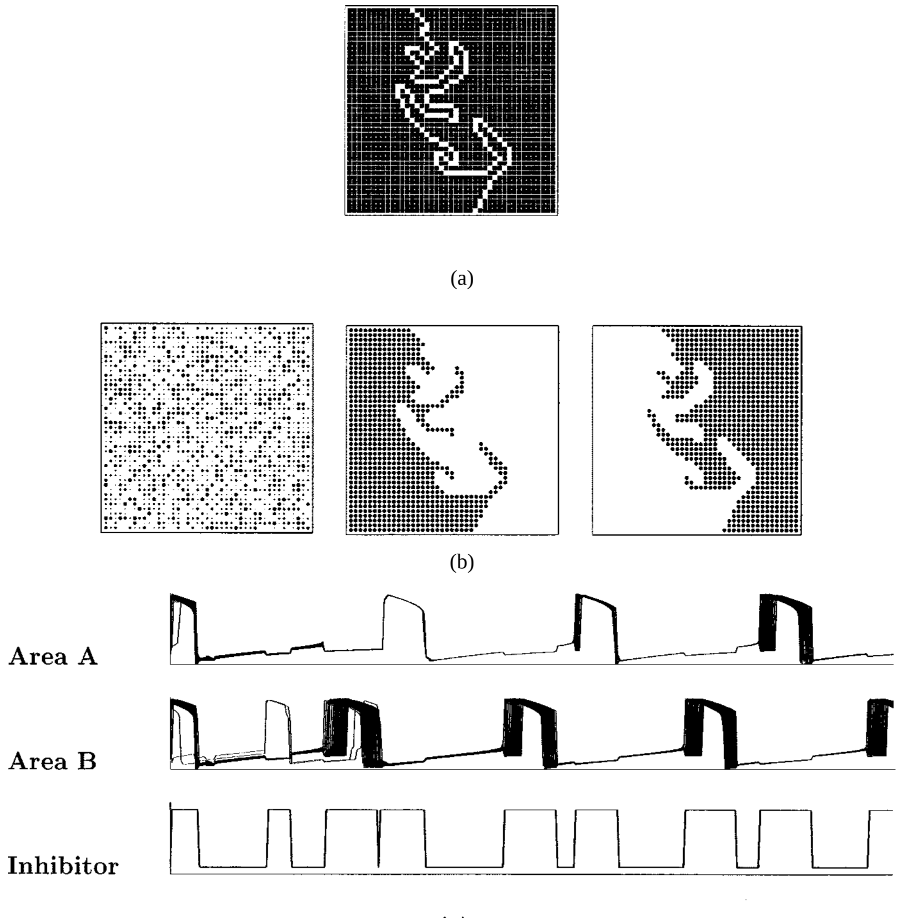


Fig. 17. Results of LEGION without time delays to the inside–outside relation of Fig. 14(a). The parameter values used in this simulation are the same as listed in the caption of Fig. 10. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing areas A and B as well as the global inhibitor, respectively. The simulation took 16 000 integration steps.

visual information processing beyond early representations. He suggested that the related computation consists of two main stages [43]. The first stage is the bottom–up creation of certain representations of the visual environment. The second stage involves serial applications of visual routines on the representations constructed in the first stage. In particular, Ullman proposed a so-called *coloring* method for computation of inside–outside relations [43]. It can be summarized as follows. Starting from a given point, its surrounding area in the internal representation is activated. The activation stretches out until a boundary is reached. Depending on the starting point, either the inside or the outside of the boundary will be activated, but simultaneous activation of both is prohibited. The coloring scheme continues until the activation cannot spread. This provides a basis of separating the inside from the outside. Our simulation results show that activity propagation in LEGION with time delays forms a traveling wave within a specific object. When an object is a spiral-like structure as in Fig. 14(b), activation spreads slowly, while activation spreads rapidly when the object is a wide open structure. This behavior of LEGION is similar to the process of the coloring scheme. However, there are a number of important differences. First, the coloring method is described as a serial algorithm, while our system is an inherently parallel and distributed process though its emergent behavior reflects a degree of serial nature of the problems. Second, the coloring method does not make a qualitative distinction between rapid effortless perception that corresponds to simple boundaries and slow effortful perception that corresponds to convoluted boundaries. In contrast, our system makes such a distinction: effortless perception with simple boundaries corresponds to when pattern formation is achieved, and effortful perception with convoluted boundaries corresponds to when pattern formation is not achieved. Third, perhaps more importantly conceptually, our system does not invoke high-level serial processes to solve such problems like inside–outside relations; its solution involves the same mechanism as it does for parallel image segmentation (see [51] and [52]). On the other hand, visual routines belong to second-stage processing, which operates on representations computed from first-stage, bottom-up processing.

From the computational standpoint, LEGION dynamics without time delays amounts to segmentation of connected components, a task that can well be solved by serial algorithms (e.g., see [29, Ch. 9]). On the other hand, LEGION is a neural network that performs segmentation in a parallel and distributed way. The LEGION dynamics described here is a simple version tailored to the problems addressed in this paper, which works on binary input and uses only four nearest

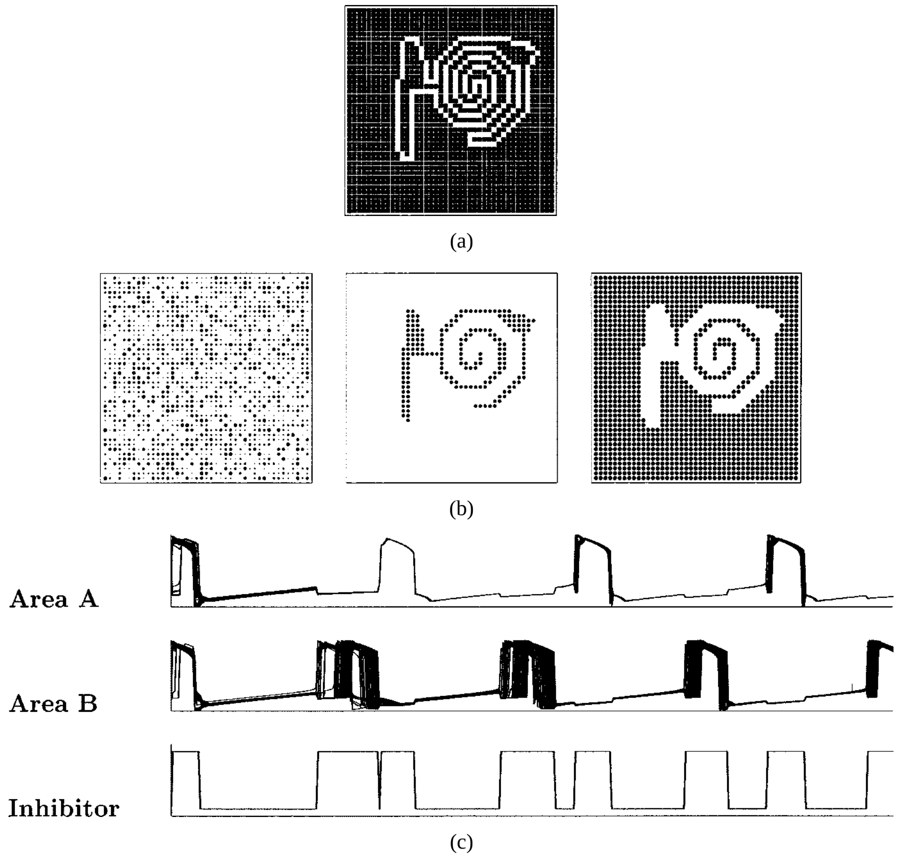


Fig. 18. Results of LEGION without time delays to the inside–outside relation of Fig. 14(b). The parameter values used in this simulation are the same as listed in the caption of Fig. 10. (a) Visual stimulus. (b) A sequence of snapshots. (c) Temporal trajectories of activities of the oscillator groups representing areas A and B as well as the global inhibitor, respectively. The simulation took 16 000 integration steps.

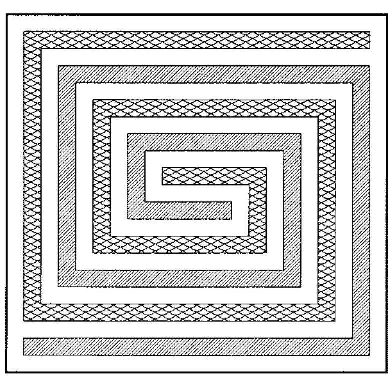


Fig. 19. Two spirals with differing textures.

neighbor coupling. Extended versions have been proposed and applied to segmenting a variety of challenging real imagery, such as medical imagery [39] and aerial imagery [6]. These studies also show favorable comparisons with other image segmentation algorithms. Because of its parallel and distributed nature and its continuous-time dynamics, LEGION has been implemented on very large scale integration (VLSI) chips for segmentation purposes [9], [1].

Finally, the style of computation performed by LEGION can be characterized as spatiotemporal processing, a topic that has been extensively studied in the literature (see, for example, [8], [24], [34]). Obviously, the LEGION networks described in this paper represent one solution in the paradigm of spatiotemporal processing. Similar solutions are possible with different oscillator models and different coupling methods.

V. Conclusion

Based on oscillatory correlation, we have given a generic solution to the spiral and inside–outside problems. Our simulation results of LEGION with time delays qualitatively resemble human performance. We find that the spiral problem and the inside–outside problem share an intrinsic property, i.e., immediate perception is unavailable if local activation cannot rapidly spread, as synchrony would not be established in the presence of time delays. As a special case, LEGION without time delays always solves these problems regardless of shape, position, size, and orientation. Given limited success from numerous attempts in the framework of learning, we suggest that such geometric problems may be better solved by a properly connected architecture with the oscillatory correlation representation than by supervised learning.

Appendix
Period of Oscillation in Legion

In this Appendix, we describe how to calculate the period of oscillation in terms of system parameters in LEGION. This calculation is similar to that of Campbell and Wang [4], which is done with time delays.

In order to calculate the period of oscillation, it is useful to explain the behavior of two coupled relaxation oscillators. Fig. 20 shows the behavior of two enabled oscillators coupled through mutual excitation. It can be regarded as the case of a network of LEGION only with two oscillators but omitting the global inhibitor. Let O_1 and O_2 denote the two oscillators. Suppose that both oscillators are initially on the lower left branch (LLB) and O_1 is closer to the lower left knee (LLK). The leading oscillator, O_1, will reach LLK first, and jump up to the lower right branch (LRB). Since O_1 has crossed a specific threshold of the interaction term θ, O_2 will receive excitation through the coupling term. Let the coupling strength be α. If O_2 is below $\mathrm{LLK}_y + \alpha$ (LLK_y is the y value of LLK), it will immediately jump up to the upper right branch (URB). When O_2 crosses the threshold θ, O_1 will receive excitation from O_2, and then hop from LRB to URB. If, however, O_2 is above $\mathrm{LLK}_y + \alpha$ when O_1 jumps up, O_2 will first hop to the upper left branch (ULB) and then jump up to URB when it is below $\mathrm{LLK}_y + \alpha$. On the other hand, O_2 will jump down to ULB when it reaches the upper right knee (URK) along URB, and then hop down to LLB when O_1 also jumps down. Each jump results in a significant phase contraction between two oscillators such that fast synchrony is achieved at an exponential rate as shown by Terman and Wang [42]. Once synchrony is achieved, the period of oscillation is the time that an oscillator traverses the periodic orbit depicted in Fig. 20.

Recently, Terman and Wang have carried out an analysis of LEGION in the singular limit ($\epsilon = 0$) and, moreover, extended their outcome to $0 < \epsilon \ll 1$ with singular perturbation analysis [42]. Basically, the process of synchronization on two coupled oscillators can be generalized to LEGION. The perturbation analysis of LEGION indicates that the exact form of x-nullcline is not important as long as a general cubic shape is maintained and evolution of the system is critically determined by solving the differential equations on the y variable [42]. When $\beta \gg 1$, we can analytically solve (1b) in terms of two different branches by utilizing $\tanh(\beta x) \approx -1$ when $x \leq -1$ and $\tanh(\beta x) \approx 1$ when $x \geq 1$. As a result, the equation describing $y_i(t)$ along one of the LBs ($x_i \leq -1$) is

$$y_i(t) = (y_i(0) - \lambda + \gamma)e^{-\epsilon t} + \lambda - \gamma, \tag{8}$$

and the y-position of an oscillator along one of the RBs ($x_i \geq 1$) is given by

$$y_i(t) = (y_i(0) - \lambda - \gamma)e^{-\epsilon t} + \lambda + \gamma \tag{9}$$

where λ and γ are parameters as defined in (1).

When synchrony is achieved in LEGION, all the oscillators representing the same object will travel along the same periodic orbit like that illustrated in Fig. 20. Here we use terms in Fig. 20 to denote the traveling path of an oscillator in LEGION. When synchrony is achieved, the time a stimulated oscillator takes to travel from LLK to URK, along URB (see Fig. 20), is calculated by (9) as

$$\tau_{\mathrm{URB}} = \frac{1}{\epsilon} \log\left(\frac{\mathrm{LLK}_y - \gamma - \lambda}{\mathrm{URK}_y - \gamma - \lambda}\right) \tag{10}$$

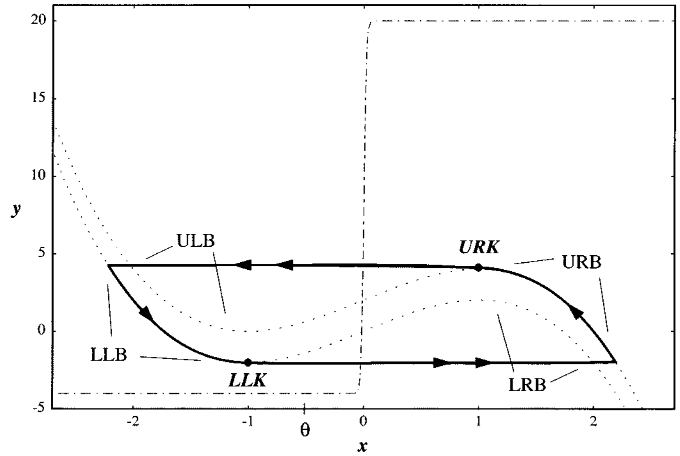


Fig. 20. Nullclines and periodic orbits of a pair of relaxation oscillators. LLK and LRK indicate the left and right knee of the lower cubic, respectively. LLB and LRB indicate the left and right branch of the lower cubic, respectively. ULK and URK indicate the left and right knee of the upper cubic, respectively. ULB and URB indicate the left and right branch of the upper cubic, respectively. See Fig. 3 caption for curve conventions.

and the time it takes to travel from URK to LLK, along LLB, is calculated by (8) as

$$\tau_{\mathrm{LLB}} = \frac{1}{\epsilon} \log\left(\frac{\mathrm{URK}_y + \gamma - \lambda}{\mathrm{LLK}_y + \gamma - \lambda}\right) \tag{11}$$

where LLK_y and URK_y are, respectively, y values of LLK and URK. Both LLK_y and URK_y will be fixed once a set of parameters in LEGION is specified; in this paper, $\mathrm{LLK}_y = -2$ and $\mathrm{URK}_y = I_i + \alpha_T - W_z + 2$ in terms of parameters as defined in (1)–(6). Therefore, the period of oscillation T is

$$T = \tau_{\mathrm{URB}} + \tau_{\mathrm{LLB}}. \tag{12}$$

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